SMLC '92-'93 w/sols

Exam #1 November 1992

1. If $\log a + 1 = b$, then $a =$

   A. 10^{b-1} B. 10^{b+1} C. $10^b - 10$ D. $10^b - 1$ E. 10^{b+1}

2. The coefficient of the x^3 term in the expansion of $(x^3 - 5x^2 + 2x + 1)(x^3 - 2x^2 - 3x)$ is

   A. 15 B. 13 C. 12 D. 11 E. 10

3. $\log_{16}(\log_2 256) =$

   A. $\frac{1}{2}$ B. $\frac{3}{4}$ C. $\frac{4}{3}$ D. $\frac{1}{4}$ E. 1

4. The graph in the xy-plane of which function below does NOT intersect the y-axis?

   A. $y = \tan x$ B. $y = \cos x$ C. $y = \sec x$ D. $y = \csc x$ E. none

5. What is the final decimal digit of the standard representation of $19^{92} - 19^{92}$?

   A. 1 B. 3 C. 4 D. 8 E. 9

6. If $(\cos t, \sin t) = \left(\frac{-3}{5}, \frac{4}{5}\right)$, then $\cos\left(\frac{\pi}{2} + t\right) =$

   A. $\frac{3}{5}$ B. $\frac{-3}{5}$ C. $\frac{4}{5}$ D. $\frac{-4}{5}$ E. none of these

7. In $\triangle ABC$, $AB = \sqrt{41}$, $BC = 13$, BD (the altitude to side $\overline{AC}$) $= 5$. The ratio of the area of the smaller region formed by the perpendicular bisector of $\overline{AC}$ to the area of $\triangle ABC$ is

   A. $\frac{1}{2}$ B. $\frac{2}{5}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$ E. none of these

8. The average age of a group of students is 22.5. If the average age of the male students is 20 and the average age of the female students is 24, then the number of male students is what fraction of the number of female students?

   A. $\frac{3}{4}$ B. $\frac{2}{5}$ C. $\frac{2}{3}$ D. $\frac{5}{3}$ E. none of these

9. The function $f(x) = \frac{x^2 - 2x + 1}{2x^2 - 1}$ has an asymptote with equation

   A. $x = 0$ B. $x = 2$ C. $x = \frac{1}{2}$ D. $y = 2$ E. $y = \frac{1}{2}$

10. The slope of the line passing through the points (2, -1) and (4, 3) in the xy-plane is

    A. 2 B. 1 C. $\frac{1}{2}$ D. 5 E. -1

11. The product of the real solutions of the equation $\frac{9}{x^2} - x^2 = 8$ is

    A. -1 B. 1 C. -9 D. 9 E. none of these

12. $\overline{AB}$ is tangent to the circle with equation $x^2 + y^2 = 1$ at the point (0,1) with B in the first quadrant. If O is the origin, C is the point of intersection of the circle and $\overline{OB}$, and $m\angle OBA = \theta$, then BC =

A. $\sec\theta$ B. $\sin\theta$ C. $\csc\theta$ D. $\sec\theta - 1$ E. $\csc\theta - 1$

13. T_1 is an equilateral triangle. T_2 is the triangle whose vertices are the midpoints of the sides of T_1, and T_{n+1} is the triangle whose vertices are the midpoints of the sides of T_n. The smallest value of n for which the area of T_n is less than $\frac{1}{1000}$ the area of T_1 is

A. 5 B. 6 C. 8 D. 10 E. 11

14. If a positive integral factor of 1992 is chosen at random, the probability it is even is

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{5}{8}$ D. $\frac{3}{4}$ E. $\frac{15}{16}$

15. $\sqrt{5 + 2\sqrt{6}} + \sqrt{5 - 2\sqrt{6}} =$

A. 10 B. $\sqrt{10}$ C. $\sqrt{11}$ D. $2\sqrt{3}$ E. none of these

16. One solution of the equation $x^4 - 4x^2 + 20x - 25$ is $1 + 2i$. Another solution is

A. $1+\sqrt{6}$ B. $2-\sqrt{6}$ C. $-1+\sqrt{6}$ D. -1 E. $-1+2i$

17. $\dfrac{1}{1 + \dfrac{2}{1 - \dfrac{1}{x}}} =$

A. $\frac{x-1}{3x-1}$ B. $\frac{x-1}{3x+1}$ C. $\frac{3x-1}{x-1}$ D. $\frac{3x-1}{x+1}$ E. $\frac{3x+1}{x-1}$

18. A game is played with nine cards numbered 1 to 9. Players alternately select and keep a card; the first to collect three cards adding to 15 wins. If your opponent (choosing first) picks 6, which of the following is your best choice to avoid losing?

A. 1 B. 2 C. 4 D. 5 E. 8

19. Let AB = 1. A point P is chosen randomly between A and B. The probability that $AP^2 + BP^2 > \frac{25}{49}$ is

A. 0 B. $\frac{6}{7}$ C. $\frac{5}{7}$ D. $\frac{\pi}{4} - \frac{25}{49}$ E. 1

20. A student is supposed to solve $x^2 + bx + c = 0$ but instead correctly solves $x^2 + cx + b = 0$ ($b \neq c$). If one of the student's solutions actually solves the original equation, what must be true about b and c?

A. $b+c=0$ B. $b-c=1$ C. $b+c=1$ D. $b-c=-1$ E. $b+c=-1$