SMLC 1993-94 w/SOLNS

Exam #1 November 1993

1. If $f(x) = x^2 - 3x$, then $f(a + h) - f(a) =$

   A. $h^2 - 3h$ B. $2ah + h^2 - 3h$ C. $2ah + h^2 - 3h - 6a$ D. $h^2 - 3h - 6a$

   E. none of these

2. A collection of nickels, dimes, and quarters worth \$2.40 has two more nickels than quarters and one more quarter than dimes. The total number of coins is

   A. 17 B. 18 C. 19 D. 20 E. 21

3. $\frac{\sin t}{\sec t}(1 + \tan^2 t) =$

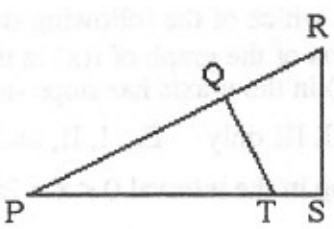
   A. $\sin t$ B. $\cos t$ C. $\csc t$ D. $\cot t$ E. $\tan t$

4. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then $A^8 =$

   A. $\begin{bmatrix} 1 & 16 \\ 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 256 \\ 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 8 & 16 \\ 0 & 8 \end{bmatrix}$ D. $\begin{bmatrix} 8 & 256 \\ 0 & 8 \end{bmatrix}$ E. none of these

5. The number of positive integers less than 1993 which are divisible by either 2 or 5 is

   A. 199 B. 996 C. 1195 D. 1394 E. 1396

6. 

   In the figure, $\angle PQT = \angle S = 90°$, $PQ = 16$, $QT = TS = 12$. Then $RS =$

   A. 16 B. 20 C. 24 D. 30 E. 32

7. For $x \neq \pm 2$, $\frac{x^2 + 2x - 8}{x^2 - 4} =$

   A. $\frac{2x - 8}{-4}$ B. $\frac{x^2 + 2x + 2}{x^2}$ C. $\frac{x - 8}{x - 2}$ D. $\frac{x + 4}{x + 2}$ E. $\frac{x - 4}{x - 2}$

8. $2(\log 12 - \log 3) =$

   A. $\log 81$ B. $\log 21$ C. $\log 8$ D. $\log 18$ E. $\log 16$

9. If AMA ÷ TYC = 6, each number is in standard form in base ten, and each letter represents a unique digit, then $(M - Y)^2 =$

   A. 0 B. 1 C. 9 D. 16 E. 25

10. In flipping four fair coins, the probability that more than half of them are heads is

    A. $\frac{4}{16}$ B. $\frac{5}{16}$ C. $\frac{6}{16}$ D. $\frac{7}{16}$ E. $\frac{8}{16}$

11. $3 + 2i$ is a zero of which of the following polynomials?

    A. $x^4 + 6x^3 + 16x^2 + 24x + 48$ B. $x^4 - 6x^3 + 16x^2 - 24x + 48$

    C. $x^4 + 6x^3 + 17x^2 + 24x + 52$ D. $x^4 - 6x^3 + 17x^2 - 24x + 52$ E. none of these

12. In a triangle with sides of length 7, 13, and 15, the angle opposite the side of length 13 measures

A. 30° B. 45° C. 60° D. 75° E. 90°

13. In ΔXYZ, $XZ = 8$, $YZ = 6$, $\angle Z = 90°$ and B and A are the points on $\overline{XY}$ which determine the angle bisector and the altitude from Z respectively. If $d = |BX - BY| + |AX - AY|$, then

A. $d < 2$ B. $2 \le d < 3$ C. $3 \le d < 4$ D. $4 \le d < 5$ E. $d \ge 5$

14. The number of distinct real solutions of $(2x^2 - 5x + 3)^{(x^3 - 3x^2 + 2x)} = 1$ is

A. 1 B. 2 C. 3 D. 4 E. 5

15. Let the xy-plane be divided into nine regions by the lines $x = \pm 2$, $y = \pm 2$. The graph of $y^2 = \frac{4x^2}{x^2 - 1}$ has points lying in how many of these nine regions?

A. 4 B. 5 C. 6 D. 7 E. 8

16. If m and n are positive integers with $m > n$, which fraction below is largest?

A. $\frac{n + 1}{m + 1}$ B. $\frac{n}{m}$ C. $\frac{n^2 - 1}{m^2 - 1}$ D. $\frac{n^2 + 1}{m^2 + 1}$ E. It depends on m and n

17. If $f(x) = mx + b$ is a linear function with slope $m \neq 0$, which of the following statements is true? (I) $f^{-1}(x)$ has slope $1/m$; (II) The reflection of the graph of $f(x)$ in the x-axis has slope $-m$; (III) The reflection of the graph of $f(x)$ in the y-axis has slope $-m$.

A. II only B. II, III only C. I, II only D. I, III only E. I, II, and III

18. The sum of all solutions of $(\sin x + \cos x)^2 = \frac{1}{2}$ lying in the interval $0 \le x < 2\pi$ is

A. 0 B. $\frac{3\pi}{2}$ C. 3π D. 4π E. 5π

19. Bill pays \$N for 7 orchids, and Sue pays \$7 for N daisies, which are cheaper than orchids. How many orchids should Bill exchange for an equal number of Sue's daisies for each of them to have a bunch of flowers of equal value?

A. 1 B. 2 C. 3 D. 4 E. 5

20. A point is chosen at random from within the part of the interior of the circle $x^2 + y^2 = 1$ in the first quadrant. The probability that at least one of the coordinates of the point is less than or equal to $\frac{1}{2}$ is closest to

A. 0.4 B. 0.5 C. 0.6 D. 0.7 E. 0.8