Test #3 **AMATYC** Student Mathematics League **March/April 1997**

1. Which of the following has as its graph a parabola which opens left?

   A. $2y^2 = 11 - 3x$ B. $5x = 13 + 2y^2$ C. $3y = 7 + 4x^2$
   D. $3x^2 = 8 - 2y$ E. $x^2 = 10 - y^2$

2. If three distinct counting numbers have a sum of 10 and a product of 20, then what is their median?

   A. 2 B. 3 C. 4 D. 5 E. There is not enough information given.

3. If L is a straight line passing through (8,11) with slope -3, then what is the y-coordinate of the intersection of L with the line $x = 5$?

   A. 2 B. 10 C. 12 D. 20 E. none of these

4. If one root of $3x^2 + j = kx$ is $2 + \sqrt{3}$, where j and k are rational, then $j + k =$

   A. -15 B. -9 C. 9 D. 15 E. none of these

5. If AB is the diameter of a circle, and C is on the circle such that $\overline{AC} = 10$ and $\overline{BC} = 16$, then the area of the circle is

   A. 89π B. $2\pi\sqrt{89}$ C. 178π D. 356π E. none of these

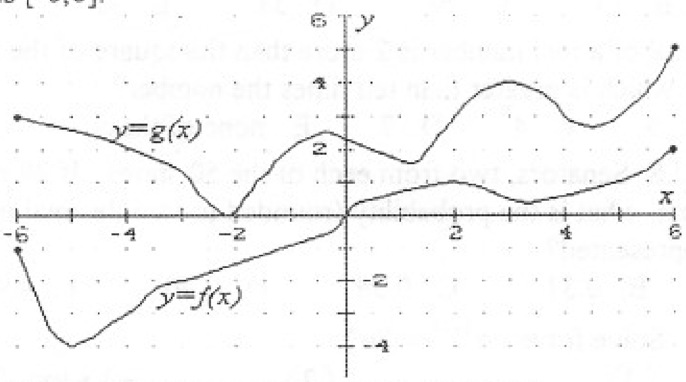
6. If the solution set for $f(x) < 3$ is $(0,+\infty]$, and the solution set for $f(x) > -2$ is $(-\infty,5)$, then the solution set for $[f(x)]^2 \geq f(x) + 6$ is

   A. $(-\infty,+\infty)$ B. $[0,5]$ C. $(-\infty,0]$ D. $[5,+\infty)$
   E. $(-\infty,0] \cup [5,+\infty)$

7. If (a,b) is a solution for $2a + 3b \leq 12$ and $5a + 2b \leq 20$, where a and b are nonnegative integers, then the maximum possible value of $a + b$ is

   A. 2 B. 3 C. 4 D. 5 E. 6

Questions 8-10 refer to the functions f and g, whose graphs are shown below and whose common domain is [-6,6].



8. How many solutions does the equation $g(x) = 2$ have?

   A. 0 B. 1 C. 2 D. 3 E. 4

9. How many solutions does the equation $g(x) = |f(x)|$ have?

   A. 0 B. 1 C. 2 D. 3 E. 4

10. Suppose that g is the restriction to [-6,6] of a periodic function G whose domain is is $(-\infty,+\infty)$ and whose period is 16. The approximate value of $G(100)$ is

    A. 0 B. 2 C. 2.3 D. 3 E. 5

11. Each of six balanced cubes is painted identically so that one letter of AMATYC is on each face (giving each cube two faces with indistinguishable A's). If the six cubes are rolled (like dice), what is the probability, rounded to two decimal places, that AMATYC can be spelled by arranging the letters that appear on the top faces of the cubes? (M, T, Y, and C must appear once; A must appear twice.)
    A. 0.03 B. 0.05 C. 0.09 D. 0.13 E. 0.17

12. A geometric sequence has a first term of $\tan t$ and a common ratio of $2\cos t$. What is the third term?
    A. $4\sin t$ B. $2\sin 2t$ C. $4\sec t$ D. $4\cos 2t$ E. $4\cos^2 t$

13. Two straight lines, whose slopes are m and n, intersect at the point (m,n), where $m \neq n$. The line of slope m has y-intercept m, and the line of slope n passes through the origin. The sum of all possible values for m is
    A. -2 B. -1 C. 1 D. 2 E. 0

14. Albert goes into a store and says "Lend me as much money as I already have, and I will spend \$20 in your store." The owner agrees, and Albert spends \$20. Albert does the same thing at a second, third, and fourth store, with the owner agreeing each time, and Albert spending \$20 each time. After this Albert has no money. What is the total of his debt to the four store owners?
    A. \$48.50 B. \$57.75 C. \$61.25 D. \$69.75 E. \$75.00

15. If $x^4 + 5x^2 - 19 = 0$, then what is the value of $x^7 - 2x^6 + 5x^5 - 5x^4 - 19x^3 + 63x^2$?
    A. 95 B. $8 + 4\sqrt{101}$ C. $12 - 4\sqrt{101}$ D. 0 E. none of these

16. Suppose all of the vehicles traveling on a certain interstate highway have either 18 wheels on 5 axles or 4 wheels on two axles. In a five minute period, 224 wheels on 88 axles pass by. How many vehicles passed by during that period?
    A. 18 B. 23 C. 29 D. 31 E. 35

17. Half the reciprocal of a real number is 2 more than the square of the number. What is the least integer which is greater than ten times the number?
    A. 1 B. 3 C. 4 D. 7 E. none of these

18. There are 100 U.S. Senators, two from each of the 50 states. If 20 senators are chosen at random, what is the probability (rounded to two decimal places) that any given state is represented?
    A. 0.29 B. 0.31 C. 0.34 D. 0.36 E. 0.40

19. Suppose $a > 0$. Solve for x: $a^{3x+1} = 2a^x$.
    A. $-1 + \log_a\left(\frac{3}{2}\right)$ B. $-1 + \log_a\left(\frac{2}{3}\right)$ C. $\frac{-1 + \log_a 2}{2}$
    D. $\log_a \sqrt{3}$ E. $\log_a \sqrt[3]{2}$

20. A picket fence has 100 vertical boards, numbered 1 through 100. Any board whose number is of the form $2k$-1 is painted red (1,3,5,7,...,99). Next any **unpainted** board whose number is of the form $3k$-1 is painted blue (2,8,14,...,98). If this process is continued, by increasing the coefficient of k by one and choosing a different color at each step, how many colors will be on the fence when every board has been painted?
    A. 26 B. 29 C. 32 D. 35 E. none of these