Test #1 **AMATYC Student Mathematics League** **October/November 2005**

1. Let $f(x) = x^2$. Find $f(2x) - 2f(x)$ for $x = -3$.

A. -18 B. -9 C. 0 D. 9 E. 18

2. The line containing the points (1,a) and (3,b) has slope -2. Find the slope of the line containing the points (1,-a) and (3,-b).

A. -2 B. $\frac{-1}{2}$ C. $\frac{1}{2}$ D. 2 E. not unique

3. The polynomial $P(x) = a_0x^{11} + a_1x^{10} + \cdots + a_{11}$ $(a_0 \neq 0)$ has at most m number of x-intercepts and at least n number of x-intercepts. The sum m + n is

A. 9 B. 10 C. 11 D. 12 E. 13

4. Jen cleans the kitchen in 20 min, her brother Ken does it in 12 min, but Ben, her two-year old brother, can mess up the kitchen in 10 min. How many minutes does it take the three of them to clean the kitchen?

A. 15 B. 20 C. 24 D. 30 E. it can't be done

5. At how many points do the graphs of $y = x^4$ and $y = 2^x$ intersect?

A. 0 B. 1 C. 2 D. 3 E. 4

6. Let M and L be two perpendicular lines tangent to a circle with radius 6. Find the area bounded by the two lines and the circle.

A. 9π B. $36 - 9\pi$ C. $144 - 36\pi$ D. 18π E. $72 - 18\pi$

7. When I am as old as my father is now, I will be five times as old as my son is now. By then, my son will be eight years older than I am now. The sum of my father's age and my age is 100 years. How much older am I than my son?

A. 14 yrs. B. 16 yrs. C. 18 yrs. D. 22 yrs. E. 24 yrs.

8. The population of Mathville grows exponentially with respect to time, and so does the number of car thefts. If f(t) represents the number of car thefts per person in Mathville with respect to time, then f(t) could NOT be

A. a constant function B. a non-constant linear function
C. an exponential growth function D. an exponential decay function
E. it could be any of these functions

9. If $a^2 - b^2 = 33$ and $a^3 - b^3 = 817$ have integer solutions with $a > b$, find the value of $a - b$.

A. 1 B. 3 C. 7 D. 10 E. 11

10. $\triangle SML$ has sides of length 6, 7, and 8. Find the exact value of (cos S + cos M + cos L).

A. $\frac{51}{35}$ B. $\frac{47}{32}$ C. $\frac{31}{21}$ D. $\frac{49}{33}$ E. $\frac{119}{80}$

11. Find the sum of all solutions of $\cos x = \cot x \cos x$ for which $0 \le x \le 2\pi$.

A. 1.5π B. 3.25π C. 3.5π D. 3.75π E. 5.5π

12. The letters of AMATYC are rearranged so that the new string starts with A, but no two letters adjacent in AMATYC are adjacent in the new string. How many such strings are there?

A. 3 B. 5 C. 6 D. 8 E. 9

13. For i = 1 to 6, let $\log_a(\log_b(\log_c x_i)) = 0$, where a, b, and c represent every possible different arrangement of 2, 4, and 8. The product $x_1x_2x_3x_4x_5x_6$ can be expressed in the form 2^N. Find N.

A. 19 B. 20 C. 28 D. 33 E. 50

14. A triangle has vertices A(0,0), B(3,0), and C(3,4). If the triangle is rotated counterclockwise around the origin until C lies on the positive y-axis, find the area of the intersection of the region bounded by the original triangle and the region bounded by the rotated triangle.

A. $\frac{21}{16}$ B. $\frac{25}{16}$ C. $\frac{29}{16}$ D. $\frac{35}{16}$ E. $\frac{75}{16}$

15. When written as a decimal number, 2005^{2005} has D digits and leading digit L. Find D + L.

A. 6623 B. 6624 C. 6625 D. 6626 E. 6627

16. If $0 < t < \pi/2$, $0 < z < 1$, and $\cos t = \frac{1 - z^2}{1 + z^2}$, how many of the following are true?

$$z = \sqrt{\frac{1 - \cos t}{1 + \cos t}};\ \sin t = \frac{2z}{1 + z^2};\ \tan t = \frac{2z}{1 - z^2};\ z = \tan\frac{t}{2}$$

A. 0 B. 1 C. 2 D. 3 E. 4

17. Let $a_1 = 2$ and $a_{n+1} = \frac{12}{2a_n + 5}$ for all $n \ge 1$. Find the value that a_n approaches as n increases without bound.

A. $\frac{3}{2}$ B. $\frac{2}{3}$ C. 12 D. 6 E. There is no such value

18. A circle contains 25 points chosen so that the arcs between any two adjacent points are equal. Three of these points are chosen at random. Let the probability that the triangle formed is right be R, and the probability that the triangle formed is isosceles be I. Find $|R - I|$.

A. $\frac{1}{5}$ B. $\frac{3}{17}$ C. $\frac{1}{7}$ D. $\frac{3}{23}$ E. $\frac{3}{25}$

19. If $x^2 + xy + 15x = 12$ and $y^2 + xy + 15y = 42$, which of the following is a possible value for x + y?

A. 3 B. 6 C. 9 D. 18 E. More than one of these

20. A point P is chosen at random inside square ABCD with AB = 1. Find the probability that all of the angles of △PAB are acute.

A. $1 + \frac{\sqrt{3}}{4}$ B. $1 + \frac{\pi}{2}$ C. $\frac{1 + \pi}{8}$ D. $1 - \frac{\pi}{8}$ E. $\frac{\pi}{4}$