Test #1 November 1993

1 B f(a+h)-f(a) = $(a+h)^2 -3(a+h)-a^2 +3a = a^2 +2ah+h^2 -3a-3h-a^2 +3a = 2ah+h^2 -3h$

2 C n = q+2 = d + 3. 5n + 10(n-3) + 25(n-2) = 240 ⇒ n = 8 , q = 6 , d = 5. Ans: 14

3 E $\frac{\sin t}{\sec t}(1+\tan^2 t) = \frac{\sin t}{\sec t}(\sec^2 t) = \sin t \sec t = \sin t \frac{1}{\cos t} = \tan t$

4 A $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$; $A^4 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$;

$A^8 = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 16 \\ 0 & 1 \end{bmatrix}$

5 C The # pos integers < 1993 which are divisible by 2 is 996. (2,4,6,...,1992=2 • 996)
The # pos integers < 1993 which are divisible by 5 is 398. (5,10,15,...,1990=5 • 398)
The # pos integers < 1993 which are divisible by 10 is 199. (10,20,30,...,1990=10 • 199)
The # pos integers < 1993 which are divisible by either 2 or 5 is 996+398-199 = 1195

6 C PT=20 by Pythagorean Thm. $\Delta PTQ \sim \Delta PRS$. $RS = \frac{PS}{PQ}QT = \frac{32}{16}\cdot 12 = 24$

7 D For $x \neq \pm 2$, $\frac{x^2+2x-8}{x^2-4} = \frac{(x+4)(x-2)}{(x+2)(x-2)} = \frac{x+4}{x+2}$

8 E $2(\log 12 - \log 3) = 2(\log(12/3)) = 2\log 4 = \log(4^2) = \log 16$

9 B AMA = 6 • TYC. T must be 1, and A must be even. A must be at least 6. Hence A is either 6 or 8. If A=6 then Y=0, and then C=1 or 6, neither of which is possible. Thus it must be that A=8. Then C=3 or 8. But C≠8. So C=3. 6(1Y3)=8M8 ⇒ 618 + 6Y = 808 + 10M ⇒ 6Y = 19 + M ⇒ Y=4,M=5. $(M-Y)^2 = (5-4)^2 = 1^2 = 1$

10 B The sample space has size 2 • 2 • 2 • 2=16. One of them has 4 heads. (HHHH). Four have three heads. (HHHT,HHTH,HTHH,THHH). Thus, the probability of 3 or more heads is $5/16$.

11 D Since the four choices listed have real coefficients, if 3+2i is a zero then so is 3-2i. Thus $(x-3-2i)(x-3+2i)=x^2 - 6x + 13$ would be a factor. Thus choices A and B are out. We observe that D is $(x^2 - 6x + 13)(x^2 + 4)$.

12 C By the Law of Cosines,

$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{7^2+15^2-13^2}{2\cdot 7\cdot 15} = \frac{49+225-169}{210} = \frac{105}{210} = \frac{1}{2} \Rightarrow B = 60°$

13 D XY=10. $\Delta XYZ \sim \Delta ZYB \Rightarrow BY = 18/5$. $AX - AY = \frac{32}{5} - \frac{18}{5} = \frac{14}{5}$.

Draw a line through B parallel to XZ, intersecting YZ at C. Then $\Delta BCY \sim \Delta XZY$.

Since BC=CZ, we have CY=6-BC. It follows that BY=$30/7$. BX-BY=$\frac{40}{7} - \frac{30}{7} = \frac{10}{7}$

$\frac{10}{7} + \frac{14}{5} = \frac{50}{35} + \frac{98}{35} = \frac{148}{35} = 4\frac{8}{35}$

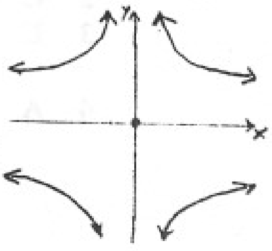
14 C Either $2x^2-5x+3=1$ or ($x^3-3x^2+2x=0$ and $2x^2-5x+3\neq 0$).

$2x^2-5x+3=1 \Rightarrow x=\frac{1}{2}$ or $x=2$.

$x^3-3x^2+2x=0 \Rightarrow x=0, x=1$, or $x=2$. However, $2x^2-5x+3=0$ when $x=1$.

Thus, there are three solutions. (0, 1/2, and 2)

15 D If y=0 then x=0. So (0,0) is on the graph.



$y\neq 0 \Rightarrow y^2>0 \Rightarrow x^2>1 \Rightarrow x\in(-\infty,-1)\cup(1,+\infty)$

For $x\in(-\infty,-1)\cup(1,+\infty)$, $y^2=\frac{4x^2}{x^2-1}=\frac{4}{1-\frac{1}{x^2}}>\frac{4}{1}=4 \Rightarrow |y|>2$

The graph has points lying in seven of the nine regions.

16 A $\frac{A}{B}=\frac{n+1}{m+1}\cdot\frac{m}{n}=\frac{mn+m}{mn+n}>1 \Rightarrow A>B$

$\frac{C}{A}=\frac{n^2-1}{m^2-1}\cdot\frac{m+1}{n+1}=\frac{(n-1)(n+1)(m+1)}{(m+1)(m-1)(n+1)}=\frac{n-1}{m-1}<1 \Rightarrow C<A$

$\frac{B}{D}=\frac{n}{m}\cdot\frac{m^2+1}{n^2+1}=\frac{nm^2+n}{mn^2+1}=\frac{m+\frac{1}{m}}{n+\frac{1}{n}}>\frac{m}{n+1}\geq 1 \Rightarrow B>D$

A is the greatest.

17 E If f(x)=mx+b, then f^{-1}(x)=(1/m)y-b/m. I is true.

If y=mx+b, then the reflection in the x-axis is (-y)=mx+b or y=-mx-b. II is true.

If y=mx+b, then the reflection in the y-axix is y=m(-x)+b or y=-mx+b. III is true.

18 E $(\sin x+\cos x)^2=\frac{1}{2} \Rightarrow \sin^2 x+2\sin x\cos x+\cos^2 x=\frac{1}{2} \Rightarrow \sin 2x=-\frac{1}{2} \Rightarrow 2x=\frac{7\pi}{6},\frac{11\pi}{6},\frac{19\pi}{6},\frac{23\pi}{6} \Rightarrow x=\frac{7\pi}{12},\frac{11\pi}{12},\frac{19\pi}{12},\frac{23}{1}$

sum = 5π

19 C Value of one orchid = N/7 ; value of one daisy = 7/N.

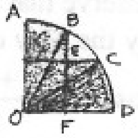
Let x = # flowers switched. Clearly $0<x<7$.

$\frac{(7-x)N}{7}+\frac{7x}{N}=\frac{Nx}{7}+\frac{7(N-x)}{N}$

Simplyfing, we obtain $2x(N+7)=7N$.

Trying the few possible values for x, we see that only x = 3 gives a positive integral value for N.

20 E P = (area of shaded region)/(area of quarter circle)



area of sector OAB = $\frac{1}{2}\cdot\frac{\pi}{6}\cdot(1)^2=\frac{\pi}{12}$

area (ΔOBE) = area(ΔOBF)-area(ΔOEF) = $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{\sqrt{3}}{8}-\frac{1}{8}$

shaded area = $\frac{\pi}{12}+\frac{\sqrt{3}}{8}-\frac{1}{8}+\frac{\sqrt{3}}{8}-\frac{1}{8}+\frac{\pi}{12}=\frac{\pi}{6}+\frac{\sqrt{3}}{4}-\frac{1}{4}$

P = $\left(\frac{\pi}{6}+\frac{\sqrt{3}}{4}-\frac{1}{4}\right)\left(\frac{4}{\pi}\right)=\frac{2}{3}+\frac{\sqrt{3}-1}{\pi}\approx .67+\frac{1.7-1}{3.14}\approx .67+.22\approx .89$