Test #2 February 1994

1 A $(2)^3+k(2)^2+k(2)+6=0 \Rightarrow 8+4k+2k+6=0 \Rightarrow 6k=-14 \Rightarrow k=-\frac{7}{3}$

2 C $3x+2=-4 \Rightarrow x=-2$. $f(-4)=(-2)^2-4(-2)=4+8=12$

3 D The sum of the original eight numbers is 8 • 32=256. The sum of the new collection is 11 • 40=440. Thus the sum of the 3 added numbers is 440-256=184, and their mean is $\frac{184}{3}=61\frac{1}{3}$.

4 E Both volumes increase by a factor of $2^3=8$.

5 A The graph has holes at $(-2, \frac{1}{8})$ and $(2, \frac{1}{8})$ but has no vertical asymptotes.

6 E a), and d) do not hold for any even function with negative values, e.g. x^2-1 at $x=0$.
b) does not hold for any even function with a zero, e.g. x^2 at $x=0$.
c) does not hold for any even function which is not also odd, e.g. x^2.
e) holds by definition

7 C Since 3 < BC < 6, a unique triangle is formed, and B is obtuse. Drop a perpendicular from B to AC, intersecting AC at D. Then BD = 3, CD=4, and AD=$3\sqrt{3}$.
Thus, AC=$4+3\sqrt{3}$.

8 A $x^2-2x+1=7 \Rightarrow x-1=\pm\sqrt{7} \Rightarrow x=1\pm\sqrt{7}$.
$\left(1+\sqrt{7}\right)^2+\left(1-\sqrt{7}\right)^2=(1+2\sqrt{7}+7)+(1-2\sqrt{7}+7)=16$.

9 C The probability that the first letter is A is 4/12. Given that the first letter is A, the probability that the second letter is M is 2/11. Given that the first two letters are AM, the probability that the third letter is A is 3/10. Given that the first three letters are AMA, the probability that the fourth letter is T is 3/9. Given that the first four letters are AMAT, the probability that the fifth letter is Y is 1/8. Given that the first five letters are AMATY, the probability that the sixth letter is C is 2/7. Thus, the probability that the six letters spell AMATYC is $\frac{4}{12}\cdot\frac{2}{11}\cdot\frac{3}{10}\cdot\frac{3}{9}\cdot\frac{1}{8}\cdot\frac{2}{7}=\frac{1}{4620}$.

10 D The graph is a "diamond" with vertices at (-10,0), (0,-5), (10,0), and (0,5).
The area enclosed is the sum of four triangles of area 25 each.

11 B

	rate	time	distance
out	r + 30	2	2r + 60
return	r - 60	3	3r - 180

$2r+60=3r-180 \Rightarrow r=240 \Rightarrow d=540$.

12 E a) and b) each have exactly 2 distinct, real zeros. The three cubic polynomials each have one as a zero. Dividing that zero out gives c) $(x-1)(x^2-2x+4)=(x-1)(x-2)^2$,
d) $(x-1)(x^2+1)$, and e) $(x-1)(x^2-5x+6)=(x-1)(x-2)(x-3)$. We see that c) has two real zero, d) has one, and e) has three.

13 E $\cos\left(\sin^{-1}\left(\frac{1}{3}\right)+\tan^{-1}(5)\right)=\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)\cos\left(\tan^{-1}(5)\right)-\sin\left(\sin^{-1}\left(\frac{1}{3}\right)\right)\sin\left(\tan^{-1}(5)\right)=$
$\frac{2\sqrt{2}}{3}\cdot\frac{1}{\sqrt{26}}-\frac{1}{3}\cdot\frac{5}{\sqrt{26}}=\frac{2\sqrt{2}-5}{3\sqrt{26}}$.

14 B a) Graph is three lines; exactly 2 pass through origin
b) Graph is three concurrent lines passing through (1,2)
c) Graph is three lines; two are parallel
d) Graph is one line

15 B The graph of each quadratic is a parabola, opening up. Thus 100 is an upper bound for the zeros if and only if the value at 100 is positive. The values of the quadratics at 100 are -89999, 100, -100, -100, and -80000 respectively.

16 D Draw the graphs, and observe the number of intersections.

17 C 7N + 11B = 348 . Find one solution by trial and error; find others using the slope. B=3, 10, 17, 24, or 31 .

18 D There are $6^3 = 216$ possibilities. The sum can not possibly be less 20 if one of the numbers rolled is 17. If one of the numbers rolled is 13, then the other two would have to be 3's. (3 possibilities: 13, 3, 3 ; 3, 13, 3 ; 3, 3, 13). If one of the numbers is 11 then the other two must be 3's (3 possibilities) or a 3 and a 5 (6 possibilities). If exactly one of the numbers is 7, then the other two must be 3's (3 possibilities), 5's (3 possibilities), or a 3 and a 5 (6 possibilities). If two of the rolls are 7's then the other must be 3 (3 possibilities) or 5 (3 possibilities). Finally if all three numbers are either 3's or 5's, the sum is less than 20 (8 possibilities). There are 38 ways to get a sum less than 20.
$\frac{38}{216} = \frac{19}{108}$

19 B
$$A^2 = \begin{bmatrix} a & b \\ c & 2a \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & 2a \end{bmatrix} = \begin{bmatrix} a^2+bc & 3ab \\ 3ac & 4a^2+bc \end{bmatrix}$$
$$A^2 + 3A = \begin{bmatrix} a^2+bc & 3ab \\ 3ac & 4a^2+bc \end{bmatrix} + \begin{bmatrix} 3a & 3b \\ 3c & 6a \end{bmatrix} = \begin{bmatrix} a^2+3a+bc & 3ab+3b \\ 3ac+3c & 4a^2+6a+bc \end{bmatrix}$$
Thus, $a^2 + 3a + 6b = -2$, $3ab + 3b = 0$, $3ac + 3c = 3$, and $4a^2 + 6a + bc = 4$.
$3ab + 3b = 0 \Rightarrow 3b(a+1) = 0 \Rightarrow b = 0$ or $a = -1$.
But $a \neq -1$, since then the second equation would not hold.
So $b = 0$ and $a^2 + 3a = -2$, $ac + c = 1$, and $2a^2 + 3a = 2$.
$a^2 + 3a = -2 \Rightarrow a^2 + 3a + 2 = 0 \Rightarrow (a+1)(a+2) = 0 \Rightarrow a = -1$ or $a = -2$.
$2a^2 + 3a = 2 \Rightarrow 2a^2 + 3a - 2 = 0 \Rightarrow (2a-1)(a+2) = 0 \Rightarrow a = 1/2$ or $a = -2$.
Thus, a must be -2 . Then $c = -1$. (Check that all four original equations are satisfied.)
$a^2 + b^2 + c^2 = (-2)^2 + (0)^2 + (-1)^2 = 4 + 0 + 1 = 5$.

20 C The second duck swam past the midpoint for one minute and then returned to the midpoint after one more minute. During these two minutes, the other ducks swam half the width of the pond (150 ft). Thus the ducks swam at a rate of 75 ft/min.