Test #1 November 1996

1 B The circumference of the circle is $C = \pi d$, where d is the diameter of the circle. But the diameter of the circle is the diagonal of the square, so $s = d/\sqrt{2}$, and the perimeter of the square is $P = 4d/\sqrt{2}$. Thus $\frac{C}{P} = \frac{\pi d}{4d/\sqrt{2}} = \frac{\pi\sqrt{2}}{d}$.

2 A $\sqrt{-t}$ is at least 10 for the given values of t. $-\sqrt[3]{t}$ is at most $\sqrt[3]{200}$, which is less than 6. $-\mathrm{Sin}^{-1}(\sin t)$ cannot possibly be greater than $\pi/2$. t^3 is always negative. And $\frac{9t-1}{t} = 9 - \frac{1}{t} < 10$.

3 E If $x>1$, then all three arguments are positive. But if $x \le 1$, $\log(x-1)$ is not real. Thus the identity is valid for (only) $x > 1$.

4 C 5, 10, 15, ..., 995 are divisible by 5. There are 995/5 or 199 of these.
7, 14, 21, ..., 994 are divisible by 7. There are 994/7 or 142 of these.
35, 70, 135, ..., 980 are divisible by both 5 and 7. There are 980/35 or 28 of these.
999-(199+142-28) = 686.

5 D $\frac{83.8(n-1)+97}{n} = 84.2 \Rightarrow 83.8n - 83.8 + 97 = 84.2n \Rightarrow 13.2 = 0.4n \Rightarrow n = \frac{13.2}{0.4} = 33.$

6 C The x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{k}{6}$. Thus the y-coordinate of the vertex is $3\left(-\frac{k}{6}\right)^2 + k\left(-\frac{k}{6}\right) + 7 = -\frac{k^2}{12} + 7$. This would be zero for $k = \pm\sqrt{84}$.

7 D If it is not true that every good boy does fine, then it must be that there is some good boy who does not do fine. That is, the negation of "every A is B" is "there is an A which is not B".

8 D $g(2) \approx 3$. $f(3) \approx 0.5$.

9 C $f(x) + g(x) = 0 \Leftrightarrow g(x) = -f(x)$. Reflect the graph of f about the x-axis and count the intersections with the graph of g.

10 A The range of g is $[0,5]$. The range of f when x is restricted to $[0,5]$ is $[0,1]$.

11 D
$$A + M + A + T + Y + C = A + M + A + \left(\frac{\pi}{2} - A\right) + \cos\left(\frac{\pi}{2} - A\right) + \left(M + \cos\left(\frac{\pi}{2} - A\right)\right) =$$
$$A + 2M + 2\sin A + \frac{\pi}{2} = A + 2M + 2M + \frac{\pi}{2} = A + 4M + \frac{\pi}{2} = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi.$$

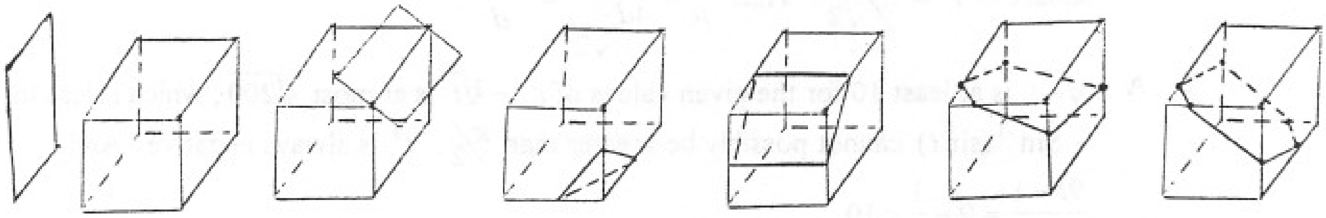
12 B $0.10_{two} = \frac{1}{2}$. $0.12_{three} = \frac{1}{3} + \frac{2}{9} = \frac{5}{9} > \frac{1}{2}$. $0.21_{five} = \frac{2}{5} + \frac{1}{25} = \frac{11}{25} < \frac{1}{2}$.
$0.42_{nine} = \frac{4}{9} + \frac{2}{81} = \frac{38}{81} < \frac{1}{2}$. $0.53_{twelve} = \frac{5}{12} + \frac{3}{144} = \frac{63}{144} < \frac{1}{2}$.

13 B BBBB, BBBG, BBGB, **BBGG**, BGBB, **BGBG**, **BGGB**, BGGG, GBBB, **GBBG**, **GBGB**, GBGG, **GGBB**, GGBG, GGGB, GGGG. $\frac{6}{16} = \frac{3}{8} = 0.375$.

14 C $x^5 = x^4 + x^3 \Rightarrow x^5 - x^4 - x^3 = 0 \Rightarrow x^3(x^2 - x - 1) = 0 \Rightarrow x = 0, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$.

15 D $2^x = 8^{y+1} \Rightarrow 2^x = 2^{3y+3} \Rightarrow x = 3y + 3$. $9^y = 3^{x-9} \Rightarrow 3^{2y} = 3^{x-9} \Rightarrow 2y = x - 9$.
$x = 3y + 3$ and $2y = x - 9 \Rightarrow y = 6$ and $x = 21 \Rightarrow x + y = 27$.

16 E



empty set | line segment | triangle | quadrilateral | pentagon | hexagon

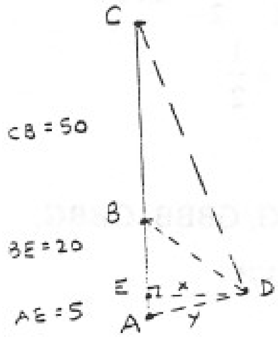
Not only are all of these six possible, there are two other possibilities for the intersection: a point and a filled square.

17 A $xy = \frac{x}{y} \Rightarrow xy^2 = x \Rightarrow x(y^2 - 1) = 0 \Rightarrow x = 0$ or $y = \pm 1$. $x = 0$ and $y = 1$ lead to contradictions. $y = -1 \Rightarrow -x = x + 1 \Rightarrow x = -\frac{1}{2} \Rightarrow x + y = -\frac{3}{2}$.

18 B The slope of the radius to (2,5) is $\frac{7-5}{6-2} = \frac{1}{2}$. Thus the slope of the tangent line is -2. Thus the equation of the tangent line is $(y - 5) = -2(x - 2)$ or $y = -2x + 9$. This line crosses the x-axis at 4.5 and the y-axis at 9. The area of the desired right triangle is $\frac{1}{2}\left(\frac{9}{2}\right)(9) = \frac{81}{4}$.

19 B The hiker heading back to the far end takes $\frac{3/5}{11} = \frac{3}{55}$ hr. The hiker heading for the near end takes $\frac{2/5}{13} = \frac{2}{65}$ hr. It takes the train $\frac{3}{55} - \frac{2}{65} \approx 0.024$ hr to travel the one mile through the tunnel. Its rate of travel is $\frac{1}{0.024} \approx 42$ mph.

20 C



Applying the Law of Sines to ΔABD, we obtain $\frac{\sin ADB}{\sin ABD} = \frac{25}{y}$.
Applying the Law of Sines to ΔBCD, we obtain $\frac{\sin BDC}{\sin CBD} = \frac{50}{CD}$.
But $\angle ADB = \angle BDC$ and $\angle ABD = \angle CBD$, so $\frac{25}{y} = \frac{50}{CD}$.
This gives $CD = 2y$. Applying the Pythagorean Theorem to ΔDEC and to ΔAED, we obtain $x^2 + 70^2 = 4y^2$ and $x^2 + 5^2 = y^2$. Subtracting gives $4875 = 3y^2$ or $y^2 = 1625$. Thus $x^2 = 1600$ and $x = 40$.