Test #1 November 1997

1 C $4s+7=1863-1776 \Rightarrow 4s+7=87 \Rightarrow 4s=80 \Rightarrow s=20$

2 D R lies between two concentric circles of radii $\sqrt{3}$ and $\sqrt{5}$. The area of R is $\pi\left(\sqrt{5}\right)^2-\pi\left(\sqrt{3}\right)^2=5\pi-3\pi=2\pi$.

3 C The square root of a product is not necessarily the product of the square roots. In fact, $\sqrt{ab}=-\sqrt{a}\cdot\sqrt{b}$ when a and b are both negative.

4 E Let x = Jake's age now. Twenty years ago, Jake's age was x - 20. The house is now 20 years old. $20=\frac{1}{2}(x-20) \Rightarrow 40=x-20 \Rightarrow x=60$.

5 A The graphs of f and f^{-1} are reflections of one another about $y=x$. Thus, for a linear function to be its own inverse, either it must be the identity function or have slope -1. Since $f(4)=10$, f is not the identity function. It follows that $f(x)=-x+14$, giving $f(9)=5$.

6 C The set contains 10 numbers. One more than the square of each number results in the following list of ten numbers: 101, 26, 10, 5, 2, 1, 2, 10, 26, 101. Since the roots of the equation are 5 and 10, we have three successes.

7 A An inscribed angle of 23° intercepts an arc of 46° on the circle. The length of this arc is thus $\frac{46}{360}$ of the circumference of the circle, or $\frac{46}{360}(18\pi)=2.3\pi\approx 7.226$.

8 B The entry of the lower left corner must be $7-n$ and the center entry must be $8-n$. It follows then that the upper right entry must be $2n$ and the lower right must be $n-1$. Adding along the right column, we have $(2n)+(7)+(n-1)=15 \Rightarrow n=3$. It is then routine to complete the square and verify that indeed we have a magic square.

9 D We observe from the graph of f that it is one-to-one and hence has an inverse. $y=x^2 \Rightarrow x=\pm\sqrt{y}$. Sketch the graph of f^{-1} by reflecting the graph of f about $y=x$, and observe that the negative sign is the correct choice. Since $f(-10)=100$ and $f(-5)=25$, it follows that $f^{-1}(x)=-\sqrt{x}$, $25\le x\le 100$.

10 A Plot the points and notice that (-4,4), (2,2), and (4,4) are collinear. No other combination of three points are collinear. $\binom{7}{3}-1=\frac{7!}{3!4!}-1=35-1=34$.

11 D Let g be the greatest of the three and l be the least of the three. Since the median is 5, the middle number is 5. We have $\frac{g+5+l}{3}=10+l$ and $\frac{g+5+l}{3}=g-15$. Solving simultaneously, we obtain $g=25$ and $l=0$. The sum of the three numbers is 30.

12 C Consider ΔOTP where O is the center of the large circle, T is a point of tangency between two of the unit circles, and P is the center of one of the unit circles passing through T. This is a right triangle with right angle T. $\angle TOP=\left(360/26\right)^\circ$.

$$\sin\left(360/26\right)^\circ=\frac{1}{r} \Rightarrow r=\frac{1}{\sin\left(360/26\right)^\circ}\approx 4.18$$