Test #3 March/April 1997

1 A B, C, and D are parabolas opening right, up, and down, respectively. E is a circle. A is a parabola opening left.

2 C $\{1,2,7\},\{1,3,6\},\{1,4,5\},\{2,3,5\}$ are the only possibilities with sum ten. Only the third set has a product of 20. The median of $\{1,4,5\}$ is 4.

3 D $y=-3(x-8)+11=-3x+35$. $y(5)=-15+35=20$.

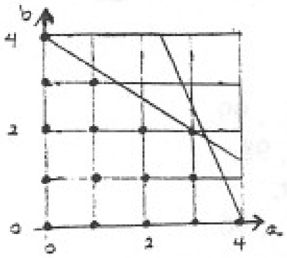
4 D Since the quadratic equation has rational coefficients, the other root must be $2-\sqrt{3}$.
$3(x-2-\sqrt{3})(x-2+\sqrt{3})=0 \Rightarrow 3(x^2-4x+1)=0 \Rightarrow 3x^2-12x+3=0 \Rightarrow 3x^2+3=12x$
Thus, $j=3$ and $k=12$. $j+k=15$.

5 A $\angle ACB$ is a right angle, so $AB=\sqrt{10^2+16^2}=\sqrt{356}=2\sqrt{89}$. Thus, the radius of the circle is $\sqrt{89}$ and its area is $\pi\left(\sqrt{89}\right)^2=89\pi$.

6 E $[f(x)]^2 \ge f(x)+6 \Leftrightarrow [f(x)]^2 - f(x) - 6 \ge 0 \Leftrightarrow (f(x)-3)(f(x)+2) \ge 0$

	$x<0$	$x=0$	$0<x<5$	$x=5$	$x>5$
$f(x)-3$	+	+	—	—	—
$f(x)+2$	+	+	+	—	—
$(f(x)-3)(f(x)+2)$	+	+	—	+	+

7 D



There are 16 solutions, as shown. The maximum value for $a+b$ is 5, occurring when $a=3$ and $b=2$.

8 E Draw the line $y=2$ and notice that it intersects the graph of g at 4 places.

9 C Draw the graph of $y=|f(x)|$ by reflecting about the x-axis the portion of the graph of $y=f(x)$ which lies below the x-axis. The graph of $y=|f(x)|$ intersects the graph of $y=g(x)$ two times.

10 D $G(100)=G(100-16\cdot 6)=G(4)=g(4)\approx 3$.

11 A If the A's were distinct, then the probability would be $\frac{6!}{6^6}$. But in fact, A_1MA_2TYC and A_2MA_1TYC are indistinguishable, so the probability is $2\cdot\frac{6!}{6^6}\approx 0.03$.

12 B The third term would be $\tan t(2\cos t)^2=4\tan t\cos^2 t=4\sin t\cos t=2\sin 2t$.

13 E The equations of the lines are $y=mx+m$ and $y=nx$. Since (m,n) is on each line, we obtain $n=m^2+m$ and $n=mn$. $n=mn \Rightarrow n-mn=0 \Rightarrow n(1-m)=0 \Rightarrow n=0$ or $m=1$. (1,2) is one possibility for (m,n). $n=0 \Rightarrow m^2+m=0 \Rightarrow m(m+1)=0 \Rightarrow m=0$ or -1. But $m \ne n$, so (-1,0) is the only other possibility for (m,n). Thus the two possibilities for m are +1 and -1, and their sum is 0.

14 C

	comes with	borrows	spends	leaves with
Store 1	x	x	20	$2x$-20
Store 2	$2x$-20	$2x$-20	20	$4x$-60
Store 3	$4x$-60	$4x$-60	20	$8x$-140
Store 4	$8x$-140	$8x$-140	20	$16x$-300

$16x - 300 = 0 \Rightarrow x = 18.75$ Since Albert spent 80, he owes 80 - 18.75 = 61.25

15 A Perform long division to obtain

$$x^7 - 2x^6 + 5x^5 - 5x^4 - 19x^3 + 63x^2 = (x^4 + 5x^2 - 19)(x^3 - 2x^2 + 5) + 95 = 0 + 95 = 95$$

16 E $18t + 4c = 224$ and $5t + 2c = 88 \Rightarrow t = 6$ and $c = 29 \Rightarrow t + c = 35$

17 B $\frac{1}{2}\cdot\frac{1}{x} = 2 + x^2 \Rightarrow 1 = 4x + 2x^3 \Rightarrow 2x^3 + 4x - 1 = 0$. There is one real solution, and it is approximately 0.2428. Thus, $10x \approx 2.428$. The least integer greater than this is 3.

18 D The probability that a given state is **not** represented is

$$\frac{{}_{98}C_{20}}{{}_{100}C_{20}} = \frac{\frac{98!}{78!\cdot 20!}}{\frac{100!}{80!\cdot 20!}} = \frac{98!\cdot 80!}{100!\cdot 78!} = \frac{80\cdot 79}{100\cdot 99} = \frac{316}{495} \approx 0.638.$$

Thus, the probability that a given state is represented is approximately 1 - 0.638 = 0.362

19 C $a^{3x+1} = 2a^x \Rightarrow \log_a\left(a^{3x+1}\right) = \log_a 2 + \log_a\left(a^x\right) \Rightarrow 3x + 1 = \log_a 2 + x \Rightarrow$
$2x = -1 + \log_a 2 \Rightarrow x = \frac{-1 + \log_a 2}{2}$.

20 A At step one ($2k$-1), we paint boards 1, 3, 5, 7, ... , 99.
At step two ($3k$-1), we paint boards 2, 8, 14, ... , 98.
At step three ($4k$-1), there are no boards to paint.
At step four ($5k$-1), we paint boards 6, 16, 26, ... , 96.
At step five ($6k$-1), there are no boards to paint.
There are only boards to paint when the coefficient of k is a prime.
The last board painted is board #100 at step one-hundred ($101k$-1).
The problem reduces to counting the number of primes less than or equal to 101.