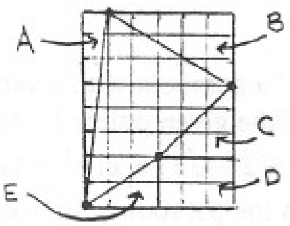
Test #2 February 1997

1 B $10^9 \text{ sec} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ yr}}{365.25 \text{ day}} \approx 31.69 \text{ yr}$

2 B $a = 2^2 \cdot 3 \cdot 5^3$, $21000 = 2^3 \cdot 3 \cdot 5^3 \cdot 7$. The least value for b is $2^3 \cdot 7 = 56$. Smaller exponents on 2 or 7 would result in an LCM less than 21000. Furthermore, the LCM of a and $2^3 \cdot 7$ is indeed 21000.

3 C $(f+g)(3) = f(3) + g(3) = 10 + 15 = 25$. Since f and g are linear, $f+g$ is linear. We observe that increasing x by 2 causes y to decrease by 5. Thus decreasing x by 2 will cause y to increase by 5. Thus, $(f+g)(1) = 25 + 5 = 30$.

4 D



$$48 - \left(\frac{1\cdot 8}{2} + \frac{5\cdot 3}{2} + \frac{3\cdot 3}{2} + 3\cdot 2 + \frac{3\cdot 2}{2}\right) = 48 - 25 = 23.$$

↑A ↑B ↑C ↑D ↑E

5 D $7^{2t+1} = 7^{2t} \cdot 7^1 = 49^t \cdot 7 = 7(49)^t \Rightarrow k = 7, a = 49 \Rightarrow k + a = 7 + 49 = 56$.

6 E The graph of f has slope $\frac{11-4}{5-3} = \frac{7}{2}$. Thus, the slope of the graph of g is $-\frac{2}{7}$.

$g(x) = -\frac{2}{7}(x-3) + 4 \Rightarrow g(2) = -\frac{2}{7}(2-3) + 4 = \frac{2}{7} + 4 = \frac{30}{7}$.

7 C Without loss of generality, let us suppose the fixed line is the x-axis and the fixed point is $(0,a)$. If (x,y) is a point satisfying the condition, then $\sqrt{x^2 + (y-a)^2} = \frac{1}{2}y$. Squaring both sides and simplifying yields $x^2 + \frac{3}{4}y^2 - 2ay + a^2 = 0$, which we see is an ellipse.

8 E Carefully translate the graph of f upward 3 units. Observe 6 intersection points with g.

9 D g has value 1 at two places: $x \approx -2.8$ and $x \approx -1.7$. f takes on each of these values exactly twice. (f takes value -2.8 at approximately -3.8 and -5.6 and takes value -1.7 at approximately -2 and -5.8). Thus there are 4 values for x giving $g(f(x)) = 1$.

10 D $3 - g(x) \geq 0 \Rightarrow g(x) \leq 3$. The longest interval for which $g(x) \leq 3$ is [-6,2], which has length 8.

11 E The slope of the given tangent line is $-\frac{3}{5}$. Thus, the slope of the radius to the point of tangency is $\frac{5}{3}$. The equation of the radius is $y = \frac{5}{3}(x-5) + 4$. Solving simultaneously with the equation of the tangent, we obtain (2,-1) for the point of tangency. The radius of the circle is $\sqrt{(5-2)^2 + (4-(-1))^2} = \sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$.

12 B Suppose the first side has been selected. Of the five remaining sides, four are adjacent to the already selected side. Thus the probability that the second selected side will have a common edge with the first selected side is $\frac{4}{5}$.

13 E Let $\theta = \text{Sin}^{-1}(\sqrt{x})$. Then $\sin\theta = \sqrt{x}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. Thus, $\cos\theta \ge 0$.

$1-\cos\left(\text{Sin}^{-1}\left(\sqrt{x}\right)\right) = 1-\cos\theta = 1-\sqrt{1-\sin^2\theta} = 1-\sqrt{1-x}$.

14 C $P = Ae^{kt}$. $2 = e^{kr} \Rightarrow kr = \ln 2 \Rightarrow k = \frac{\ln 2}{r} \Rightarrow P = Ae^{(t\ln 2/r)} = A(2)^{t/r}$.

$3 = 2^{t/r} \Rightarrow \ln 3 = \frac{t}{r}\ln 2 \Rightarrow t = \frac{r\ln 3}{\ln 2}$

15 C [The problem is flawed in that it should have stated "a parabola with a vertical axis". Otherwise, the parabola need not go through any of the given points.] Assuming that the axis of the parabola is vertical, $y = k(x-2-\sqrt{3})(x-2+\sqrt{3}) = k(x^2-4x+1)$, where k can be determined by using the fact that $(0,10)$ is on the parabola. $10 = k(+1) \Rightarrow k = 10$. $y = 10\left(x^2-4x+1\right)$. $y(3) = 10(9-12+1) = 10(-2) = -20$. None of the others check.

16 A The smallest angle θ is opposite the smallest side. By the Law of Cosines, $3^2 = 5^2+6^2-(2)(5)(6)(\cos\theta) \Rightarrow 9 = 61-60\cos\theta \Rightarrow \cos\theta = \frac{52}{60} = \frac{13}{15}$. Thus,

$\theta = Cos^{-1}\left(\frac{13}{15}\right)$, which is disguised as answer A.

17 D $8^{3x+1} = 4^{x^2-2} \Rightarrow 2^{9x+3} = 2^{2x^2-4} \Rightarrow 9x+3 = 2x^2-4 \Rightarrow 2x^2-9x-7 = 0$, which has 2 roots, the sum of which is $9/2$.

18 B Any reasonable estimate of a penny's thickness will distinguish the correct answer. In fact, it takes about 18 pennies to make an inch. 10! pennies $= \frac{10!}{18}$ inches $= \frac{10!}{18\cdot 12}$ ft $=$

$\frac{10!}{18\cdot 12\cdot 5280}$ miles ≈ 3.2 miles

19 B $n+d+q = 53$ and $5n+10d+25q = 705 \Rightarrow n+d+q = 53$ and $n+2d+5q = 141 \Rightarrow$ $d+4q = 88$. We observe that d must be a multiple of 4. The possibilities are

n	1	4	7	10	13	16	19	22	25	28	31
d	40	36	32	28	24	20	16	12	8	4	0
q	12	13	14	15	16	17	18	19	20	21	22

n certainly cannot be 11. Some made a good case for 31 also being an impossible value, depending on the interpretation of "consists entirely of nickels, dimes, and quarters." We came close to throwing the question out, but in the end ruled that B is the "best" answer.

20 C The car is traveling 5 mph faster than the train. It will take 1/5 hour for the car to overtake the mile long train, or 12 minutes.