Test #3 March/April 1998

1 E Plot the given vertices and observe the approximate location of the fourth vertex. To get from (-4,-3) to (-1,4), go right 3 and up 7. Following the same directions from (1,-2), arrive at (4,5).

2 C $m = \dfrac{4-(-3)}{8-5} = \dfrac{7}{3}$. $-3 = \dfrac{7}{3}(5) + b \Rightarrow b = -\dfrac{44}{3}$. $m + b = \dfrac{7}{3} - \dfrac{44}{3} = -\dfrac{37}{3}$

3 D Plot the points and observe that the data is approximately linear. Use linear regression or estimate the slope directly from sample points.

4 A Replace x with x+2: $f((x+2)-1) = f(x+1) = (x+2)^2 - 3(x+2) + 5 = x^2 + x + 3$.

5 B $\dfrac{3}{8}x = \dfrac{4}{5}x + \dfrac{2}{7} \Rightarrow x = -\dfrac{80}{119}$. $3\left(-\dfrac{80}{119}\right)^3 - \dfrac{3}{7} \approx -1.34$, which lies in (-2,-1).

6 D There are 7! seating possibilities, so it will take 7! days to exhaust the possibilities. 7! days = 5040 days ≈ 13.8 years.

7 C Figure n has $2n + 1$ unit squares in its middle row. The number of unit squares in figure 100 is $201 + 2(1+3+5+\ldots+199) = 201 + 2\left(\dfrac{100}{2}(1+199)\right) = 201 + 20{,}000 = 20{,}201$.

8 C The graph of f is a parabola, and the axis of symmetry is $x = 5$. So $f(4) = f(6)$, $f(-2) = f(12)$, and $f(10) = f(0) \neq 0$. The fraction quickly reduces to 1.

9 E $\begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix}^2 = \begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix} \cdot \begin{bmatrix} a & -2 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a^2-2 & -2(a+d) \\ a+d & d^2-2 \end{bmatrix}$. Thus $a^2 = d^2 = 3$, and $a = -d$. Since $a > 0$, it follows that $d < 0$. Thus $d = -\sqrt{3}$.

10 B There are 11 intersection points. Graph and count carefully. 4 of the 11 points are on the boundary of the rectangular region.

11 A B has infinitely many vertical asymptotes. C has none. D has one ($x = 3$). E has one ($x = 1$). A has two ($x = -2$ and $x = 3$).

12 D Using the identity $\cos 2A = 2\cos^2 A - 1$, we have $\cos\left(2\,\mathrm{Cos}^{-1} x\right) = 2\cos^2\left(\mathrm{Cos}^{-1} x\right) - 1 = 2\left(\cos\left(\mathrm{Cos}^{-1} x\right)\right)^2 - 1 = 2x^2 - 1$.

13 A $49^{3\log_7 4} = \left(7^2\right)^{3\log_7 4} = 7^{6\log_7 4} = \left(7^{\log_7 4}\right)^6 = 4^6 = 2^{12}$.

14 B For $c = 0$ there are 21 possibilities. For $c = 1$ there are 20 possibilities. For $c = 2$ there are 19 possibilities. ... For $c = 20$ there is one possibility.

$21 + 20 + 19 + \ldots + 1 = \dfrac{21}{2}(21+1) = \dfrac{21}{2}(22) = 231$.

15 B Create a coordinate system with C=(0,0), A=(6,0), and B=(0,6). Then the equation of AE is $y = -\dfrac{2}{3}x + 4$, and the equation of BD is $y = -\dfrac{3}{2}x + 6$. Solve simultaneously to obtain that F = (2.4,2.4). $\text{area}(\Delta BDA) = \dfrac{1}{2}(2)(6) = 6$. $\text{area}(\Delta FDA) = \dfrac{1}{2}(2)(2.4) = 2.4$.

Thus $\text{area}(\Delta BFA) = 6 - 2.4 = 3.6$.

16 D The five complex zeros of P are 2, -1/2, -1/2, 1 + 2i, and 1 - 2i. There are four distinct complex zeros. Their product is $(2)(-1/2)(1+2i)(1-2i) = (-1)(5) = -5$.

17 E Use the Law of Cosines on ΔABC to get $AB = \sqrt{5^2 + 15^2 - 2(5)(15)\cos 120^\circ} = 5\sqrt{13}$.

Use the Law of Sines on ΔACD and ΔBCD to get $\frac{AD}{\sin 60^\circ} = \frac{5}{\sin \angle ADC}$ and $\frac{BD}{\sin 60^\circ} = \frac{15}{\sin \angle BDC}$. But $\sin \angle ADC = \sin \angle BDC$, so $BD = 3AD$ and $AD = \frac{5}{4}\sqrt{13}$.

Use the Law of Sines on ΔABC to get $\frac{\sin A}{15} = \frac{\sin 120^\circ}{5\sqrt{13}} \Rightarrow \sin A = \frac{3\sqrt{3}}{2\sqrt{13}}$..

Use the Law of Sines on ΔACD to get

$$\frac{CD}{\sin A} = \frac{5\sqrt{13}/4}{\sin 60^\circ} \Rightarrow CD = \left(\frac{2}{\sqrt{3}}\right)\left(\frac{5\sqrt{13}}{4}\right)\left(\frac{3\sqrt{3}}{2\sqrt{13}}\right) = \frac{15}{4} = 3.75 .$$

18 B The center of the ellipse is (0.5,4). The distances from (-2,2) to the two foci are $\sqrt{5}$ and $2\sqrt{5}$, respectively. Thus $2a = 3\sqrt{5}$ and $a = \frac{3}{2}\sqrt{5}$. Since $c = 1.5$, we have $b = \sqrt{a^2 - c^2} = \sqrt{11.25 - 2.25} = \sqrt{9} = 3$. Thus the maximum y-value will occur 3 units above the center. $4 + 3 = 7$.

19 A The diameter of a circle with area one is $\frac{2}{\sqrt{\pi}}$. The radius of the circumscribing circle is $\frac{1}{2} + \frac{2}{\sqrt{\pi}}$. Thus the area of the circumscribing circle is $\pi\left(\frac{1}{2} + \frac{2}{\sqrt{\pi}}\right)^2 \approx 8.33$.

20 C The solution set for $x^2 + 3x < 10$ is (-5,2), an interval of length 7. The numbers in this interval which are also solutions for $x^2 > 5$ are those in the interval (-5,-$\sqrt{5}$), whose length is $-\sqrt{5} - (-5) = 5 - \sqrt{5}$. The desired probability is thus $\frac{5 - \sqrt{5}}{7}$.