1 C -5 is not a solution; all of the others are solutions.

2 C $x = -3y^2 - 12y - 16 = -3(y+2)^2 - 4$ parabola, opening left, vertex at (-4,-2)

3 D $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} j & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2j+1 & 0 \\ 3j-1 & -10 \end{bmatrix}$. Thus $2j+1=7$ and $3j-1=k$. So $j=3$, $k=8$, and $3j+5k = 3\cdot 3 + 5\cdot 8 = 9+40 = 49$.

4 B A penny is about 0.75" in diameter. The distance from NY to CA is about 2500 miles. 2500 mi = $2500 \cdot 5280 \cdot 12 = 158{,}400{,}000$ inches. It takes 158,000,000/0.75 pennies to cover the distance, or about \$2,000,000. Any reasonable estimates of the diameter and the distance will distinguish this choice from the others.

5 A Since the mode is 51, that value must occur at least twice. Since the median is 83, 51 cannot occur three times (else the median would be 51). So the two smallest numbers are 51 and 51, and the third number averages with 51 to give 83. So the third number is 115. Since the mean is 107, the sum of the four numbers is 428, giving us 211 for the fourth number. The range is 211 - 51 = 160.

6 D $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos(x) - \sin\left(\frac{3\pi}{2}\right)\sin(x) = (0)(\cos(x)) - (-1)(\sin(x)) = \sin(x)$

B. and C. are equal, since the cosine function is even, and are both also equal to $\sin(x)$.

$\sin(2\pi - x) = \sin(-x) = -\sin(x) \neq \sin(x)$

$\sin(\pi - x) = \sin(\pi)\cos(x) - \cos(\pi)\sin(x) = (0)(\cos(x)) - (-1)(\sin(x)) = \sin(x)$

7 B The lowest score obtainable is -10 (all 20 incorrect) and the highest score is 40 (all 20 correct), giving 101 potential scores. 0 correct and x incorrect shows that all of the potential nonpositive scores occur. x correct and 0 incorrect shows that all of the even integer possibilities occur. x correct and 1 incorrect shows that all of the possibilities that are a half more than an odd integer occur, up through 37.5. x correct and 2 incorrect shows that all of the possibilities that are odd integers occur, up through 35. x correct and 3 incorrect show that all of the possibilities that are half more than an even integer occur, up through 32.5. But the following six potential values do not occur: 39.5, 39, 38.5, 37, 36.5, and 34.5. 101 - 6 = 95.

8 A If 3 is the zero of a linear function, it follows that $f(3+t) = -f(3-t)$ for all t. Thus, $f(-2)+f(8)=0$, $f(1)+f(5)=0$, and $f(6) = -f(0)$. The fraction quickly collapses to -1.

9 B There are $2^6 = 64$ possibilities. To spell AMATYC, we need the first card to be M, since that's the only M, and the sixth card to be T, since we can't use the B. But then the second card would have to be A, since we can't use 2 T's. So from the third, fourth, and fifth cards we need AYC in any order. ACY and YAC are the only possibilities. So only two of the 64 possibilities will enable us to spell AMATYC. 2/64 = 1/32.

10 B $x = -2y^2 - 8y - 3 = -2(y+2)^2 + 5 \Rightarrow$ parabola with vertex at (5,-2).

$x^2 + 2y^2 - 4y = 3 \Rightarrow x^2 + 2(y-1)^2 = 5 \Rightarrow$ ellipse with center (0,1).

$d = \sqrt{(5-0)^2 + (-2-1)^2} = \sqrt{25+9} = \sqrt{34}$

11 C Let $P(x) = k(x-a)(x-b)(x-c)(x-d)$ and $Q(x) = l(x-a)(x-b)(x-c)(x-e)$. Then $\frac{P(x)}{Q(x)} = \frac{k(x-d)}{l(x-e)}$, $x \neq a,b,c$. The graph is a hyperbola with asymptotes $y=1$ and $x=e$ and three deleted points.

12 B B is a third quadrant angle, since its sine and cosine are negative, with related angle B - π. $\text{Cos}^{-1}\left(-\frac{2}{7}\right)$ is a second quadrant angle, with related angle $\pi - \text{Cos}^{-1}\left(-\frac{2}{7}\right)$. Since the two related angles must be equal, we obtain $\text{Cos}^{-1}\left(-\frac{2}{7}\right) = 2\pi - B$.

13 D Let k be the middle entry. Then the upper left entry must be 11-k, the lower center entry must be 14-k, and the right center entry must 15-n-k. This gives us two ways to calculate the upper right entry. $3+k = n+k-4 \Rightarrow n = 7$. Complete the square and verify.

14 B Let x be the number of 32-cent stamps and y be the number of 23-cent stamps. Then $32x + 23y = 691$. x and y must be nonnegative integers and y must be odd. 21 and 29 are upper bounds for x and y, respectively. The only solution is $x = 18$, $y = 5$. A calculator with a table feature is very helpful.

15 E Sketch the graph of f and observe that f is a one-to-one function, taking on value 10 for some $x < -3$. $2^{-x} - 3 = 10 \Rightarrow 2^{-x} = 13 \Rightarrow -x = \log_2 13 \Rightarrow x = -\log_2 13$.

16 D Substitute $y = 2x$ into $x^2y + xy^2 + 3xy + 5x - 7y = 5$ and simplify to obtain $6x^3 + 6x^2 - 9x - 5 = 0$, which has three distinct real roots. There are three intersection points located at roughly (1.1,2.2), (-0.45,-0.9), and (-1.5,-3).

17 E Draw AC giving isosceles right triangle ADC with $AC = 10\sqrt{2}$. $\angle ACB = 130° - 45° = 85°$ and $\angle CAB = 118° - 45° = 73°$. Apply the Law of Sines to ΔABC to obtain $\frac{\sin 22°}{10\sqrt{2}} = \frac{\sin 73°}{BC}$. Thus $BC = \frac{(10\sqrt{2})(\sin 73°)}{\sin 22°} \approx 36.1$.

18 C $x + 2y = k$ has slope $-\frac{1}{2}$. Since the tangent line to a circle is perpendicular to the radius, the radius to the point of tangency must have slope +2. Since the center of the circle is the origin, the radius would have to lie along the line $y = 2x$. That gives two possibilities for the point of tangency. $x^2 + (2x)^2 = 9 \Rightarrow x^2 = \frac{9}{5} \Rightarrow x = \pm\frac{3}{\sqrt{5}}$. $k = \pm\frac{3}{\sqrt{5}} \pm \frac{12}{\sqrt{5}} = \pm 3\sqrt{5}$.

19 A The radius of the circle with area one is $\frac{1}{\sqrt{\pi}}$. The radius of the circumscribing circle is the hypotenuse of a right triangle with one leg equal to $\frac{1}{\sqrt{\pi}} + 1$ and the other leg equal to $\frac{1}{2}$.

The area of the circumscribing circle is thus $\pi\left(\sqrt{\left(\frac{1}{\sqrt{\pi}}+1\right)^2 + \left(\frac{1}{2}\right)^2}\right)^2 \approx 8.47$.

Note: The problem statement should include that each point of tangency is at a center of a side of the square.

20 D The probability of n people NOT having a birthday today is $\left(\frac{364}{365}\right)^n$. So the probability there IS a birthday today is $1 - \left(\frac{364}{365}\right)^n$. $\left(\frac{364}{365}\right)^n = \frac{1}{2} \Rightarrow n = \frac{\ln 0.5}{\ln\left(\frac{364}{365}\right)} \approx 252.65$.

The desired probability will be greater than 0.5 when $n \geq 253$.